A NOVEL APPROACH TO THE BOUND MUON DECAY RATE RETARDATION

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ABSTRACT

Via quantum mechanics, we show that, just like the gravitational field, the electric field too slows down the internal mechanism of a clock, which enters into interaction with the field. This approach perfectly explains the retardation of the decay of muon bound to a nucleus.

For a *"real"* atomistic or molecular wave-like object, i.e. a wave-like object existing in nature, we have shown elsewhere[1] the following theorem, *first*, on the basis of the Schrodinger Equation, as complex as this may be, *then* on the basis of the Dirac Equation, whichever may be appropriate, in relation to the *frequency* of the internal dynamics of the object in hand. A *"real"* atomistic and molecular wave-like object, involves a *potential energy* made of only "*Coulomb Potential energies"*. Thence even a *relativistic Dirac description* embodying potential energies made of potential energies other than Coulomb Potentials energies, may not represent a *"real"* description.

Theorem 1 : In a *"real wave-like description" (thus, not embodying artificial potential energies)*, composed of I electrons and J nuclei, if the *(identical)* electron masses m_{i0}, i = 1,..., I, and different nuclei masses m_{j0}, j = 1,…, J, belonging to the object, are overall multiplied by the *arbitrary number* γ, then *concurrently, a)* the *total energy* E_0 associated with the given internal motion of the object, is increased as much, *and b)* the *size* R_0 to be associated with this motion contracts as much; in *mathematical words* this is

$$\{ [(m_{i0},\ i = 1,...,\ I) \rightarrow (\gamma m_{i0},\ i = 1,...,\ I)],\ [(m_{j0},\ j = 1,\ldots,\ J) \rightarrow (\gamma m_{j0},\ j = 1,\ldots,\ J)] \} \Rightarrow \{ [E_0 \rightarrow \gamma E_0],\ [\mathsf{R}_0 \rightarrow \frac{R_0}{\gamma}] \} . \quad (1)$$

Next we define a quantity called the clock mass M_0; it is the *compound mass* carrying the internal dynamics of the object; it is manufactured based on different masses embodied by the object in hand; thus multiplying these masses by γ, alters M_0 just as much.

Eq.(1) immediately yields the invariance of the quantity $E_0 M_0 \mathsf{R}_0^2$. This is *remarkable,* since this quantity is as well Lorentz invariant *(were the object brought into a uniform translational motion).*

The elements composing the quantity $E_0 M_0 \mathsf{R}_0^2$ anyway are all, somewhat *quantized quantities.* Therefore $E_0 M_0 \mathsf{R}_0^2$ ought to be in relation with a *Lorentz invariant, universal constant,* incorporated by the wave-like description in question; thus we show[1] that the quantity $E_0 M_0 \mathsf{R}_0^2$ is *"strapped"* to the square of the Planck Constant, h^2 *(being proportional to it, through a rather complex, dimensionless, and relativistically invariant quantity, which is somewhat a characteristic of the bond structure of the wave-like object in hand).*

We call this occurrence, the UMA (Universal Matter Architecture) Cast.

Note that *primarily* what we do is not a "*dimension analysis*"; $E_0 M_0 \mathsf{R}_0^2$ would anyway not be invariant in regards to a mass change, if the wave-like object in question were not *"real*", though of course, *dimension-wise* there would still be no problem.

Our finding further holds for nuclear wave-like objects embodying a potential term made of Yukawa Potentials.[1]

Anyhow, it ought to, since, as we just pointed out, the quantity $E_0 M_0 \mathsf{R}_0^2$ happens to be *Lorentz invariant,* which makes that the *special theory of relativity,* stringently imposes an *interrelation* in between E_0, M_0 and R_0^2 *(and this already at rest),* which is precisely the proportionality of $E_0 M_0 \mathsf{R}_0^2$ to h^2.

The mass increase we introduced above, may very well be not hypothetical, and this is indeed what one experiences, when a clock is removed out of a gravitational field; its rest mass, following our claim, according to the special theory of relativity,[2] would be increased as much as its original binding energy to the gravitational body of concern *(just like the mass of the hydrogen atom is increased as the electron is removed away from the proton).* The unit time of the object in hand, were this a wave-like clock, according to our quantum mechanical findings, stated above, should then be altered as much. Strikingly, this is also what happens in the scope of the general theory of relativity.[2]

Yet according to our approach, the same phenomenon would occur, in exactly the same way, for ionized wave-like clocks in an electric field, or for wave-like clocks bearing an electric dipole, still in an electric field, or for wave-like clocks bearing a magnetic diplole in a magnetic field.[3]

Similarly if the a muon is bound to a proton, its half life would quantum mechanically increase as much as its binding energy. This happens, to our knowledge, something totally overlooked.

CALCULATION OF THE MUON DISINTEGRATION HALF LIFE

Keeping *temporarily* aside the *relativistic effect* due to *(had we assumed so)* the rotation *of* the muon around the nucleus, based on Theorem 1, we can write

$$T = \frac{T_0}{\left(1 - \frac{E_B}{m_0 c^2}\right)}; \qquad (2)$$

in this relationship T_0 and T represent the decay half lives of respectively the *free muon* and that of the *bound muon;* E_B is the binding energy of the muon to the nucleus of concern.

m_0, should be the *mass of the free muon,* supposing that, the negative electric charge of the muon is distributed uniformly to its entire mass, and that, the muon internal dynamics is altered accordingly, when bound to a nucleus. However this may not be true. Indeed what is bound to the positively charged nucleus, should most likely be the *"muon's electron",* and not the *"muon"* as a whole. This muonic electron most likely pulls, the *neutrino* and the *antineutrino,* together with itself, to the binding state. Thence m_0 should be considered as the *highly energetic electron's mass inside the muon*.

Note that there seems to be six different channels of decay of the muon.[4] So the *constituents* of the muon *(supposing that these, acquire their identities inside the muon, at least, prior to the decay)*, should really depend on these channels. The one we just considered, is the main decay channel.

Now, we can express E_B *(the binding energy of the muon)* for the ground state, based on the Bohr-Sommerfeld, or here the same, the general Dirac Model, with the familiar notation;

$$E_B = \frac{2\pi^2 m_{0\mu} Z_0^2 e^4}{h^2}\left(1+\frac{1}{4}\alpha^2 Z_0^2\right); \tag{3}$$

$m_{0\mu}$ is the muon's rest mass, Z_0 the atomic number of the nucleus binding the muon, h the Planck Constant, e the electron's charge; α, is the *fine structure constant.*

The electron's mass in the free muon can be expressed as [$f m_{0\mu} c^2$], f following our claim, being 0.5. (Thus $0.5\, m_{0\mu}$ is the *effective mass of the electron,* responsible of the binding of the muon.)

α is

$$\alpha = \frac{2\pi e^2}{ch} = \frac{1}{137}. \tag{4}$$

The denominator γ, of Eq.(2), thus becomes

$$\gamma = 1 - \frac{E_B}{f m_{0\mu} c^2} = 1 - \frac{1}{2f}\alpha^2 Z_0^2\left(1+\frac{1}{4}\alpha^2 Z_0^2\right), \quad f = 0.5\,. \tag{5}$$

Next, we have to take into account the time dilation due to the rotation v of the muon around the nucleus *(had we presumed so);* this is

$$\beta = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \cong \frac{1}{\sqrt{1-\frac{4\pi^2 Z_0^2 e^4}{h^2 c^2}}}\;; \tag{6}$$

here v the rotational speed of the muon in consideration; it is evaluated through the Bohr-Sommerfeld approximation, which should be expected to be quite satisfactory for light nuclei; for heavy nuclei, quantum effects must be expected to come into play, and it is pointed that, Eq. (6) stays as an approximation all the way through.[5]

Anyway, the overall decay half life T of the bound muon, through Eqs. (2), (3), (4) and (5), quite satisfactorily, becomes

$$T = \frac{T_0}{\left[1-\frac{1}{2f}\alpha^2 Z_0^2(1+\frac{1}{4}\alpha^2 Z_0^2)\right]\sqrt{1-\alpha^2 Z_0^2}}\;. \tag{7}$$

(for the muon bound to the ground state) .

It is interesting to note that this expression does not depend on the muon's mass.

If the electron did bear any internal mechanism, the above expression would also tell us how its internal mechanism slows down, when the electron is in a bound state. (f, in this case, should be taken as unity.)

CHECK AGAINST EXPERIMENTAL AND PREVIOUS THEORETICAL RESULTS

In regards to preexisting experimental results, we were totally uninformed. We received (in June the 2nd, 1999) from the British Library two articles,[4,5] and we are more than happy to discover that our prediction about the bound muon decay, matches quite well with the experimental results. Moreover our prediction appears to be much better than any other prediction made so far, no matter how sophisticated, also inevitably cumbersome this may be.

The predictions in question, handle the retardation of the decay process through i) a semiclassical approach, which embodies the *"phase space effect" (consisting in the reduction of the volume of phase space of the muon decay products, because of the binding status of the muon),* the classical *"relativistic time dilation effect",* and *"the electron Coulomb effect" (consisting in the attraction exerted by the binding nucleus on the muonic electron)* , and ii) sophisticated quantum mechanical approaches.

It would be interesting to compare quickly our prediction ($_{\text{Author}}$) [cf. Eq.(5)], with the semiclassical (sc) results, embodying no time dilation effect.

$$\gamma_{SC} \cong 1 - \frac{11}{2}\alpha^2 Z_0^2 \quad \text{(for light } Z_0\text{)} \tag{8}$$

$$\gamma_{SC} \cong 0.58\left(1 - \alpha^2 Z_0^2\right)^{5/2} \quad \text{(for heavy } Z_0\text{)} \tag{9}$$

$$\gamma_{\text{Author}} \cong 1 - \alpha^2 Z_0^2\left(1 + \frac{1}{4}\alpha^2 Z_0^2\right) \cong 1 - \alpha^2 Z_0^2 \quad \text{(for all } Z_0\text{)} \tag{10}$$

Other predictions are so complicated that, they bear no easy series expansions.

In Figure 1 we present the experimental data and the results of previous calculations *(decay rate normalized to the decay rate of the free muon, versus the atomic number),* achieved to explain these data. Curve A is a semiclassical calculation including the time dilation effect. Curve B is the same for a Gaussian muon wave function. Curve C is a semiclassical calculation of the time dilation effect alone. Curve D is an interpolation from an anterior calculation achieved by Gilinsky and Mathews.[6] Curve E is interpolated from the calculations achieved by Huff.[6] The experimental results are achieved by Yovanovitch, Barrett, Holmstrom, Keufel, Lederman and Weinrich.[7, 8, 9]

In Figure 2 we present our prediction, as the denominator of the RHS of Eq.(7), versus the atomic number, together with the corresponding data in hand. We also sketch separately, γ of Eq. (5), versus the atomic number, since this constitutes the basis of our claim.

CONCLUSION

On the whole, clearly, our prediction is much better than other predictions. It embodies a totally different philosophy. It is surprizingly simple, whereas other predictions are quite complex. It is also amazing to note that we came to predict the retardation of the decay of bound muons, through our original quantum mechanical approach, which as well yields the end results of the general theory of relativity *(and this, without having to assume the authentic "principle of equivalence").*[3]

Thus excitingly enough we come to state that just like *"mass"; "electric charge"* too, slows down interacting clocks.

Note that the data embody a peak near iron. Our approach did not predict it. Yet neither could the previous attempts. It is suspected that this may be due to the *large background of low energy gamma rays* associated with accompanying *inelastic muon capture* events.

The usefulness of our approach, whenever it can be considered to be valid *(as discussed throughout),* consists in avoiding any worries emerging from *internal mechanistic complications of the bound muon*, which indeed requires very sophisticated tools to deal with.[6,7]

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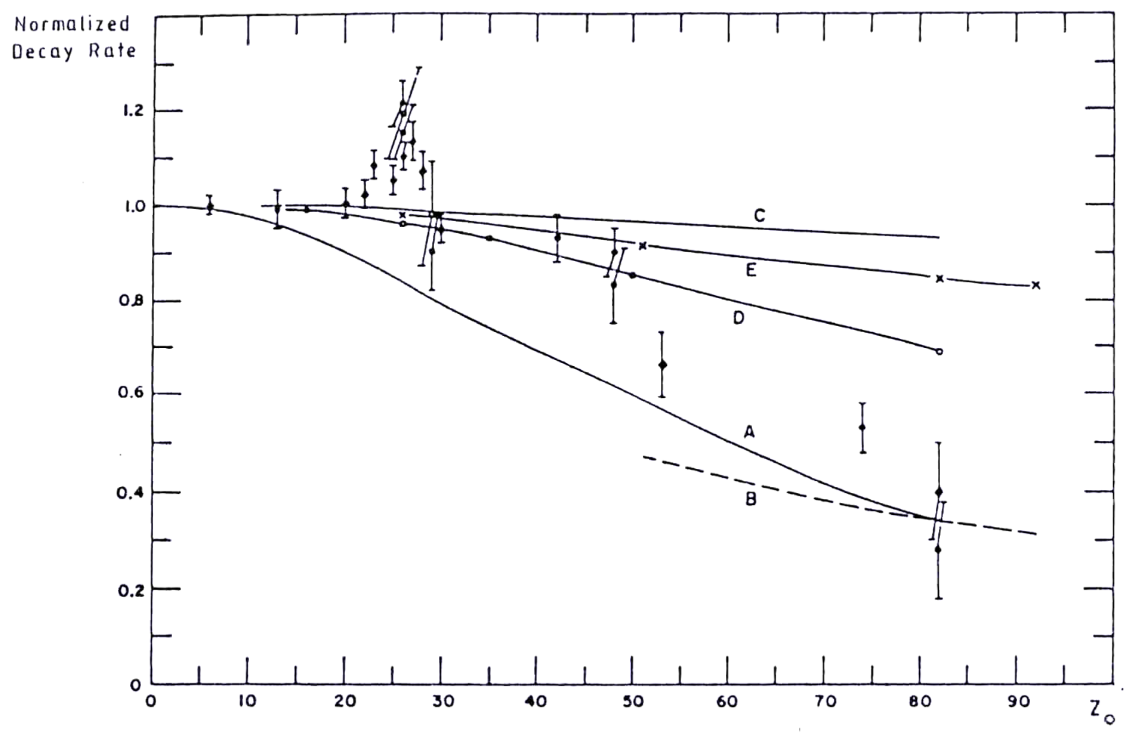

Figure 1 Preexisting experimental results and theoretical predictions [36]

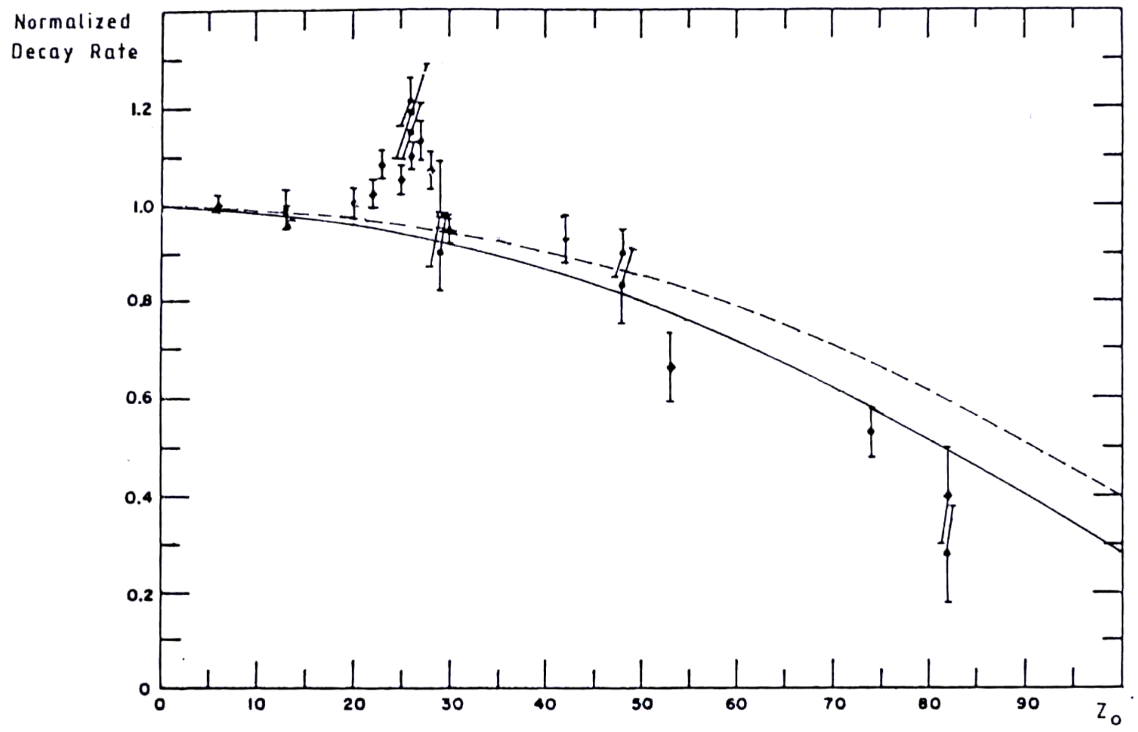

Figure 2 Preexisting experimental results and our predictions:

——— : the overall decay rate

- - - - : the decay rate without the relativistic time dilation effect

REFERENCES

[1] T. Yarman**, Invariances Based on Mass and Charge Variation, Manufactured by Wave Mechanics, Making up The Rules of Universal Matter Architecture,** Chimica Acta Turcica, Vol.27, 1999.

[2] A. Einstein, **The Meaning of Relativity,** Princeton University Press, 1953.

[3] T. Yarman, **A Novel Approach to The End Results of The General Theory of Relativity and to Bound Muon Decay Retardation,** DAMOP 2001 Meeting, APS, May 16-19, 2001, London, Ontario, Canada.

[4] F. Herzog, K. Adler, **Decay Electron Spectra of Bound Muons,** Helvetica Physics Acta, Vol. 53 (1980).

[5] R. W. Huff, **Decay Rate of Bound Muons,** Annals of Physics, 16: 288-317 (1961).

[6] V. Gilinsky, J. Mathews, Phys. Rev. 120, 1450 (1960).

[7] D. D. Yovanovitch, Phys. Rev. 117, 1580 (1960).

[8] W. A. Barrett, F. E. Holmstrom, J. W. Keufel, Phys. Rev. 113, 661 (1959).

[9] L. M. Lederman, M. Weinrich, Proceedings of the CERN Symposium on High-Energy Accelerators and Pion Physics, Geneva, 1956, Vol. 2 (427).